

Statistics

Fall 2022

Lecture 14



Feb 19-8:47 AM

College **claims** that **less than 45%** of **all** students are in favor of face-to-face classes. $\rightarrow P < .45$

In a **sample of 625** students, **44%** of them were in favor of face-to-face classes.

use $\alpha = .02$ to test the claim. $x = n\hat{p} = 625(.44) = 275$

$H_0: P \geq .45$ CV \geq LTT $\alpha = .02$

$H_1: P < .45$ claim, LTT

CTS $Z = -.503$
P-value $P = .308$

1-Prop Z Test

$P_0: .45$
 $x = 275$
 $n = 625$
Prop $< P_0$

$Z = \text{invNorm}(.02, 0, 1)$

CTS is in NCR $\Rightarrow H_0$ valid
P-value $> \alpha \Rightarrow H_1$ invalid

Invalid claim \Rightarrow **Reject the claim**

Nov 29-6:50 PM

LA Times has reported that the mean age of all nurses in So. CAL. is 45 yrs.
 $H_0: \mu = 45$ claim

In a sample of 40 nurses, their mean age was 50 yrs.
 $n = 40$ $\sigma = 15$
 $\bar{x} = 50$

It is also known that standard deviation of ages of all nurses in So. CAL. is 15 yrs.

Determine the validity of LA times report.

NO α
 \Rightarrow Use .05

Since σ known
 Z-Test

CV Z TTT $\alpha = .05$

H_1 CR .025 H_0 NCR .95 H_1 CR .025

CTS $Z = 2.108$

P-value $P = .035$ ✓ $Z = \text{invNorm}(.975, 0, 1)$

$\mu_0 = 45$
 $\sigma = 15$
 $\bar{x} = 50$
 $n = 40$
 $\mu \neq \mu_0$

CTS is in CR H_0 invalid
 $P\text{-value} \leq \alpha \Rightarrow H_1$ valid
 Invalid claim \Rightarrow **Reject the claim**

If we change α to .01, .02, or .03 then
 $P\text{-value} > \alpha \Rightarrow H_0$ valid \rightarrow Valid claim
 FTR the claim

Nov 29-6:59 PM

Given CTS $Z = 2.108$, TTT

Find P-value.

$P\text{-value} = \text{Area} * 2$

$\mu = 0$ $\sigma = 1$ 2.108

$P\text{-value} = 2 * \text{normalcdf}(2.108, E99, 0, 1)$
 $= \boxed{.035}$

Nov 29-7:10 PM

12 exams were randomly selected, here are the Scores:

75	83	94	100
66	70	80	90
58	72	85	95

Find
 1) $\bar{x} = 81$ Round to whole
 2) $S = 13$

I claim the mean of all exams is more than 78. $\mu > 78$ $H_0: \mu \leq 78$
 $H_1: \mu > 78$ claim, RTT

Use $\alpha = .1$ to test the claim.
 Since σ is unknown
 CV t RTT $\alpha = .1$
 $df = n - 1 = 12 - 1 = 11$

CTS $t = .799$
 P-value $P = .220$ ✓
 $t = \text{invT}(.9, 11)$

T-Test
 $\mu_0 = 78$
 $\bar{x} = 81$
 $S = 13$
 $n = 12$
 $\mu > \mu_0$

CTS is in NCR $\Rightarrow H_0$ valid
 $P\text{-value} > \alpha$ H_1 invalid
 Invalid claim \Rightarrow Reject the claim

Nov 29-7:13 PM

Given CTS $t = .799$, RTT, $df = 11$
 Find P-value.

$P\text{-value} = \text{Area}$

$= \text{tcdf}(.799, E99, 11)$ $\mu = 0$
 σ unknown
 $df = 11$

$= .221$ ✓

Nov 29-7:24 PM

Testing one population standard deviation σ :

$H_0: \sigma = \sigma_0$	$H_0: \sigma \leq \sigma_0$	$H_0: \sigma \geq \sigma_0$
$H_1: \sigma \neq \sigma_0$	$H_1: \sigma > \sigma_0$	$H_1: \sigma < \sigma_0$
TTT	RTT	LTT

Always identify the claim $\hat{=}$ Type of testing

Use p-value method only

CTS $\chi^2 = \frac{(n-1) \cdot S^2}{\sigma^2}$

use $\chi^2_{cdf}(L, U, df)$ to find p-value.

Proceed with testing chart

Final Conclusion must be about the claim

Nov 29-7:27 PM

Given: $n=10$, $S=22$, $H_0: \sigma \leq 20$

claim is H_0 , $\alpha = .04$

Test the claim.

$H_0: \sigma \leq 20$ claim

$H_1: \sigma > 20$ RTT

CTS $\chi^2 = \frac{(n-1) \cdot S^2}{\sigma^2}$

$$\chi^2 = \frac{(10-1) \cdot 22^2}{20^2} = 10.89$$

Area = P-value

$$= \chi^2_{cdf}(10.89, E99, 9) = .283$$

$P\text{-value} > \alpha \Rightarrow H_0 \text{ valid}$
 $.283 > .04 \Rightarrow H_1 \text{ invalid}$

valid claim
 FTR the claim

Nov 29-7:32 PM

College claims that standard deviation of ages of all students is at least 10 yrs.

$H_0: \sigma \geq 10$ claim

$H_1: \sigma < 10$ LTT


I randomly selected 20 students, and standard deviation of their ages was 8.5 yrs.

$n=20$ $S=8.5$

Test the claim.

No $\alpha \Rightarrow .05$

CTS $\chi^2 = \frac{(n-1) \cdot S^2}{\sigma^2} = \frac{(20-1) \cdot 8.5^2}{10^2} = 13.728$



$P\text{-value} = \chi^2_{cdf}(0, 13.728, 19) = .201$

$P\text{-value} > \alpha \Rightarrow H_0 \text{ valid} \rightarrow \text{Valid claim}$
 $.201 > .05$ $H_1 \text{ invalid}$ (FTR the claim)

Nov 29-7:38 PM

Dept. of health services claims that standard deviation of salaries of all nurses is \$400 per month.

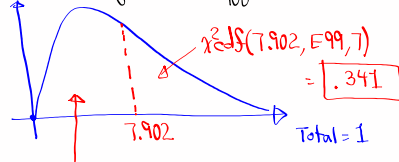
$H_0: \sigma = 400$ claim

$H_1: \sigma \neq 400$ TTT

I took a sample of 8 nurses stand. dev. of their salaries was \$425 per month.

Use $\alpha = .01$ to test the claim. $n=8$ $S=425$

CTS $\chi^2 = \frac{(n-1) \cdot S^2}{\sigma^2} = \frac{(8-1) \cdot 425^2}{400^2} = 7.902$



$\chi^2_{cdf}(7.902, \infty, 7) = .341$

$\chi^2_{cdf}(0, 7.902, 7) = .659$

$P\text{-value} = 2 \cdot \text{Smaller area} = 2 \cdot (.341) = .682$

$P\text{-value} > \alpha \Rightarrow H_0 \text{ valid} \rightarrow \text{Valid claim}$
 $.682 > .01$ $H_1 \text{ invalid}$ (FTR the claim)

SG 26 & 27

Nov 29-7:46 PM

[SG 31]

Comparing Two Population Standard Deviations:

Sample 1	Sample 2	$\sigma_1 \neq \sigma_2$
$n_1 =$	$n_2 =$	Ndf = $n_1 - 1$
$s_1 =$	$s_2 =$	Ddf = $n_2 - 1$

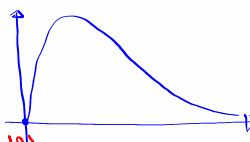
$s_1 > s_2$

$H_0: \sigma_1 = \sigma_2$	$H_0: \sigma_1 \leq \sigma_2$	$H_0: \sigma_1 \geq \sigma_2$
$H_1: \sigma_1 \neq \sigma_2$	$H_1: \sigma_1 > \sigma_2$	$H_1: \sigma_1 < \sigma_2$
TTT	RTT	LTT

CTS $F = \frac{s_1^2}{s_2^2}$

Use Fcdf to find P-value

Fcdf(L, U, Ndf, Ddf)



F-Dist.

We can also use 2-Samp F Test or CTS $\hat{=}$ P-Value

Proceed with testing chart (P-value method)

Final conclusion about the claim

Reject the claim OR FTR the claim

Nov 29-8:12 PM

Use the chart below

Sample 1	Sample 2	1) verify that $s_1 > s_2$
$n_1 = 8$	$n_2 = 10$	✓
$s_1 = 12$	$s_2 = 10$	2) Ndf = $n_1 - 1 = 7$ Ddf = $n_2 - 1 = 9$

3) Use $\alpha = .1$ to test the claim that $\sigma_1 = \sigma_2$.

$H_0: \sigma_1 = \sigma_2$ claim

$H_1: \sigma_1 \neq \sigma_2$ TTT

[STAT] [TESTS] [↓] → 2-Samp F Test

CTS $F = \frac{s_1^2}{s_2^2} = \frac{12^2}{10^2}$ ↓

CTS F = 1.44
P-Value P = .597

Inpt: [stats]

$s_1 = 12$
 $n_1 = 8$
 $s_2 = 10$
 $n_2 = 10$

P-Value $>$ α

.597 $>$.1

H_0 valid → Valid claim $\sigma_1 = \sigma_2$

H_1 invalid → FTR the claim

Nov 29-8:20 PM

10 Female students had stand. dev. of 8
for their ages. $n=10, S=8$

12 Male students " " " " = 5
for their ages. $n=12, S=5$

Females	Males
$n_1=10$ Ndf=9 $S_1=8$	$n_2=12$ Df=11 $S_2=5$

$S_1 > S_2$
Test the claim that there is a difference between two Pop. standard deviations.

$H_0: \sigma_1 = \sigma_2$

$H_1: \sigma_1 \neq \sigma_2$ claim, TTT NO $\alpha \rightarrow$ use .05

CTS $F = 2.56$ ← CTS $F = \frac{S_1^2}{S_2^2} = \frac{8^2}{5^2} = 2.56$

P-value $P = .144$

P-value $>$ α
.144 $>$.05

2-Samp F Test

H_0 valid, H_1 invalid

Stat TESTS

Invalid claim

Reject the claim

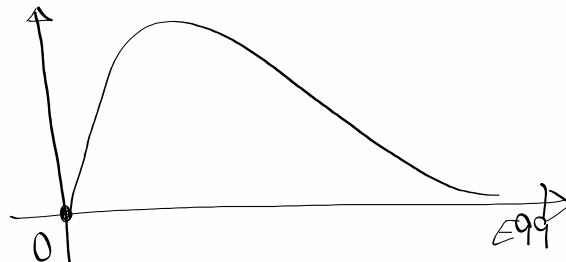
Nov 29-8:27 PM

F-dist

- Graph is similar to χ^2 -dist
starts at 0, skewed to the right,
Not symmetric, Total Area = 1

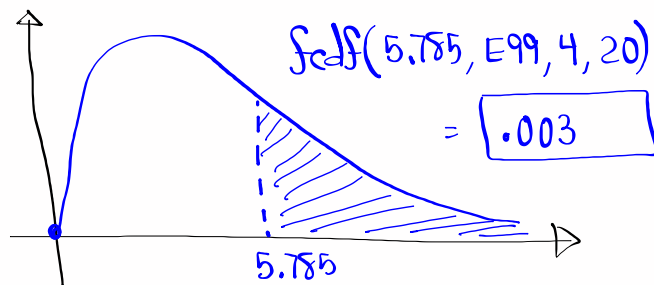
- Numerator df \rightarrow Ndf
Denominator df \rightarrow Ddf

- use $Fcdf(L, U, Ndf, Ddf)$

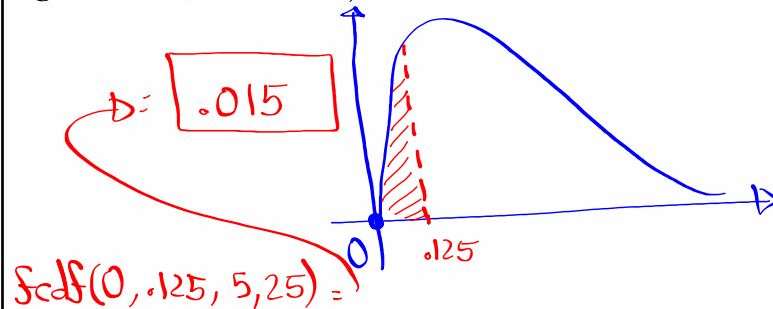


Nov 29-8:37 PM

Find $P(F > 5.785)$ with $Ndf=4, Ddf=20$.

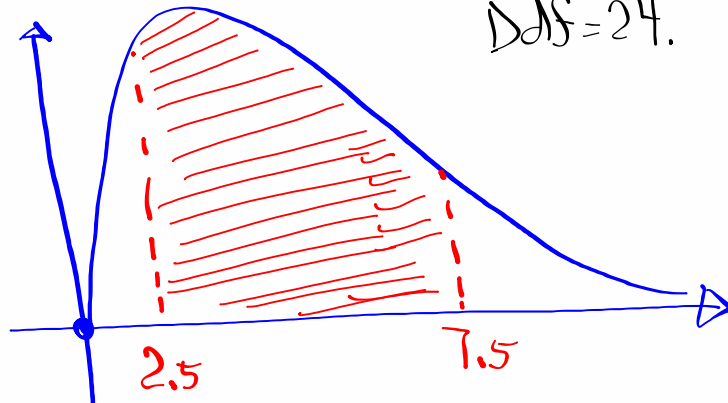


Find $P(F < .125)$ with $Ndf=5, Ddf=25$



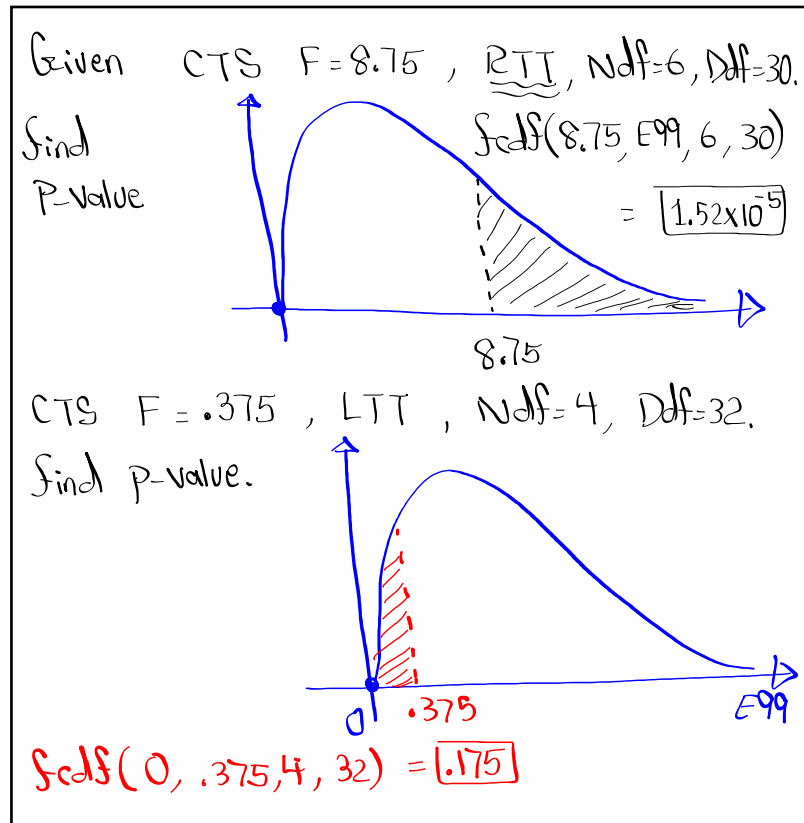
Nov 29-8:39 PM

Find $P(2.5 < F < 7.5)$ with $Ndf=3, Ddf=24$.

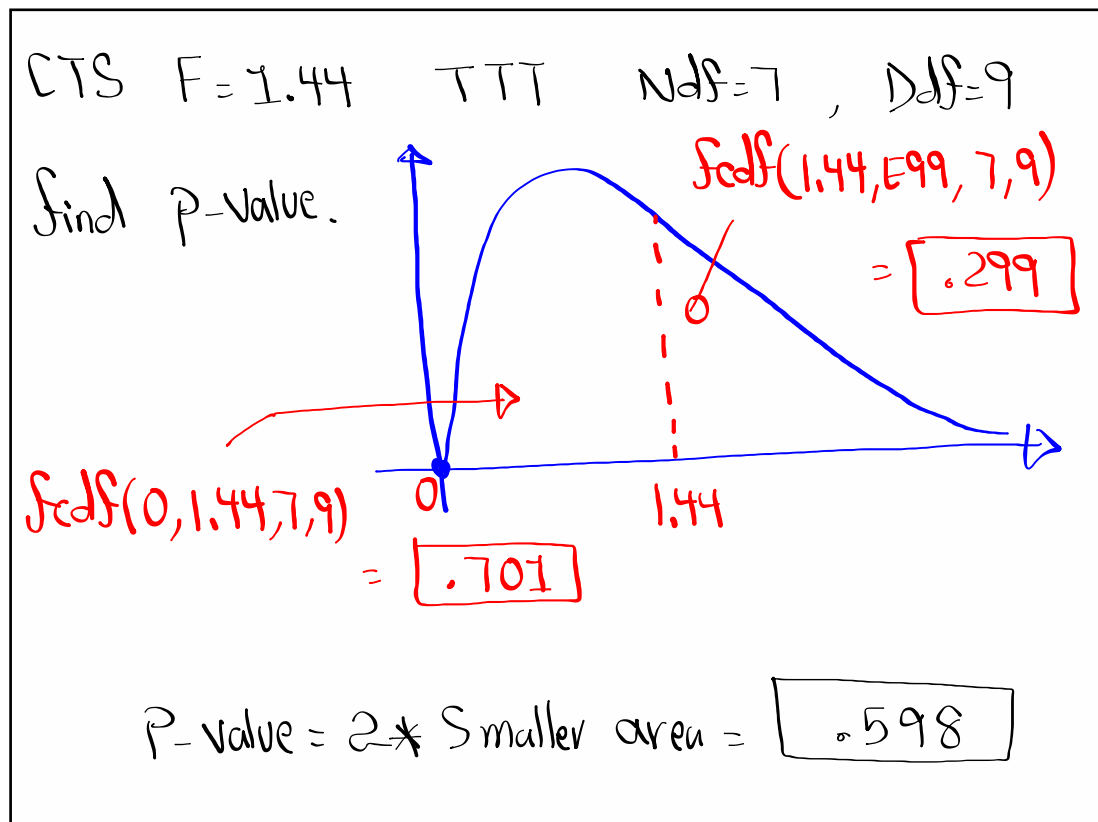


$$f_{cdf}(2.5, 7.5, 3, 24) = .083$$

Nov 29-8:43 PM



Nov 29-8:46 PM



Nov 29-8:50 PM

Morning class	Evening class
72 85 90	70 86 94
65 100 80	69 100
$\bar{x} = 82$	$\bar{x} = 84$
$S = 13$	$S = 14$
$n = 6$	$n = 5$

Round to whole #

Test the claim that $\sigma_1 = \sigma_2$. \rightarrow NO α Use .05

Sample 1	Sample 2
$n_1 = 5$	$n_2 = 6$
$S_1 = 14$	$S_2 = 13$

CTS $F = 1.160$
P-value $P = .854$ ✓
2-Samp F Test
P-value $> \alpha$
.854 $>$.05
 H_0 is valid
 H_1 is invalid
Valid claim
FTR The claim

$H_0: \sigma_1 = \sigma_2$ claim
 $H_1: \sigma_1 \neq \sigma_2$ TTT

Ndf = $n_1 - 1 = 4$
Ddf = $n_2 - 1 = 5$

Nov 29-8:54 PM

Given CTS $F = 1.160$, Ndf = 4, Ddf = 5
TTT Find p-value.

$Fcdf(1.160, E99, 4, 5) = .427$

$Fcdf(0, 1.160, 4, 5) = .573$

P-value = 2 * Smaller area
 $= 2 (.427) = .854$

Final Exam in 2 weeks.
Start Your Preparation.

SG 31 ✓

Nov 29-9:01 PM